GRAVITATIONAL CONICAL BREMSSTRAHLUNG AND DIFFERENTIAL STRUCTURES

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Differential properties of a spin 2 boson field $\psi_{\mu\nu}$ describing propagation of gravitational perturbations on a straight cosmic string's space-time background are studied by means of methods of the differential spaces theory. It is shown that this field is a smooth one in the interior of cosmic string's space-time and looses this property at the singular boundary except for cosmic string space-times with the following deficits of angle: $\Delta = 2\pi(1 - 1/n)$, $n = 1, 2, \ldots$.

A relationship between smoothness of $\psi_{\mu\nu}$ at the singularity and the gravitational conical bremsstrahlung effect is discussed. A physical interpretation of the smoothness notion is given. It is also argued that the assumption of smoothness of $\psi_{\mu\nu}$ at the singularity plays an equivalent role to the Aliev and Gal'tsov "quantization" condition.

1 Gravitational Conical Bremsstrahlung Effect

Conical bremsstrahlung appears when a point mass is moving in a space-time with the conical singularity. Then the particle produces perturbations of the background metric propagating in form of gravitational waves. The effect was found by Aliev and Gal'tsov [1,2] and is an example of gravitational version of the Aharonov-Bohm effect; radiative phenomena appear in absence of local gravitational forces. The total radiative energy emitted by a particle moving near the straight cosmic string's conical singularity vanishes for the following special values of the deficit of angle: $\Delta = 2\pi(1 - 1/n), n = 1, 2, \ldots$. The same effect holds for a K-G scalar and an electromagnetic fields.

2 Differential Spaces and the Smoothness Notion

Every space-time M as a Lorentzian manifold is endowed with various mathematical structures. Among other ones, there is a Sikorski's *differential structure* $\mathcal{C}$. Briefly speaking, it is a family of real functions on M such that: a) $\mathcal{C}$ is closed with respect to localization, b) $\mathcal{C}$ is closed with respect to superposition with smooth functions on $\mathbb{R}^n$ for any $n \in \mathbb{N}$ and c) M is equipped with topology such that every function from $\mathcal{C}$ is continuous [3].

The pair $(M, \mathcal{C})$ is said to be a *differential space* and is a notion more general than the manifold one; every manifold is a d-space but not every d-space is a manifold. For example, the space-time of straight cosmic string is a d-space which is

a manifold in contrast to the cosmic string's space-time with the conical singularity which is not a manifold but is still a d-space.

In the theory of d-spaces the smoothness notion plays the fundamental role. It is a generalization of the well known one from the manifold theory. Thus, a real function $\phi : M \to \mathrm{I\!R}$ is said to be a smooth one on $(M, \mathcal{C})$ if $\phi \in \mathcal{C}$ while a vector field $\mathbf{V} : \mathcal{C} \to \mathcal{C}$ is said to be smooth one on $(M, \mathcal{C})$ if $\mathbf{V}(\mathcal{C}) \subset \mathcal{C}$.

The smoothness definition for tensor fields is somewhat more complicated [3,4]. But, for the tensor potential $\psi_{\mu\nu}$ important in this paper the test of smoothness is based (generally speaking) on verification of the following condition:

$$\psi_{\mu\nu} V^\mu V^\nu \in \mathcal{C},$$

for every smooth vector field $\mathbf{V} : \mathcal{C} \to \mathcal{C}$. In the complex case, a complex tensor field is a smooth one if its real and imaginary parts are smooth real tensor fields.

3   Smoothness of Gravitational Perturbations

Gravitational perturbations of the straight cosmic string's space-time produced by a moving particle are described by the potential $\psi_{\mu\nu}$ satisfying in the De-Donder gauge $\nabla_\mu \psi^{\mu\nu} = 0$ the following field equation: $\nabla_\mu \nabla^\mu \psi_{\rho\sigma} = 0$. The elementary solutions of the wave equation are denoted by $\psi_{\mu\nu}^{(\epsilon\beta l)}$.

> **Proposition:** *The elementary solutions $\psi_{\mu\nu}^{(\epsilon\beta l)}$ are smooth complex tensor fields on the cosmic string space-time manifold.*

Thus, the elementary solutions have the smoothness property characteristic for the theory of differential spaces. The space time of cosmic string with singularity is not a manifold but is still a d-space. Can the smoothness property of $\psi_{\mu\nu}^{(\epsilon\beta l)}$ survive after prolongation to singularity?

> **Theorem:** *The elementary solutions $\psi_{\mu\nu}^{(\epsilon\beta l)}$ after prolongation to singular boundary are smooth complex tensor fields on the differential space of cosmic string with singularity only for the following deficits of angle $\Delta = 2\pi(1 - 1/n), n = 1, 2, ...$ For the remaining deficits of angle they are not smooth complex tensor fields.*

The proof is based on verification of the condition mentioned in the previous section. The similar theorem holds for a scalar and an electromagnetic fields [5,6].

4   Discussion

The family of all straight cosmic string's space-times can be divided into two separate subfamilies. For the first one ($\Delta = 2\pi(1 - 1/n), n = 1, 2, ...$) every elementary perturbation $\psi_{\mu\nu}^{(\epsilon\beta l)}$ is a smooth complex tensor field on the whole cosmic string's space-time background including singularity. For the second one ($\Delta \neq 2\pi(1 - 1/n), n = 1, 2, ...$) not every elementary perturbation has the smoothness property. According to the results of Aliev and Gal'tsov mentioned in the

Section 1, this division is also natural from strictly physical view point. Namely, the conical bremsstrahlung effect existing for the second subfamily vanishes for the first one. The mathematical reason of this coincidence are differential properties of the elementary solutions at singularity (smoothness). They are mirrored in the Aliev and Gal'tsov results through the radiative Green function constructed with help of $\psi_{\mu\nu}^{(\epsilon\beta l)}$.

The above coincidence enable the following physical interpretation of the smoothness notion. Every massive body moving in the gravitational field of cosmic string emits, in general, gravitational waves. One can expect that such situation will continue until the deficit of angle reaches one of the distinguished values: $\Delta = 2\pi(1 - 1/n), n = 1, 2, ...$ Then, the system (a particle moving in gravitational field of cosmic string) becomes stationary; the conical bremsstrahlung effect vanishes. One can express the same in terms of strictly geometrical notions from the theory of d-spaces. The system will radiate till every non smooth elementary mode becomes smooth one. The nonsmoothness of the elementary solutions indicates that the system is not stationary. The system becomes stationary if every elementary solution $\psi_{\mu\nu}^{(\epsilon\beta l)}$ is a smooth tensor field on the whole space-time of cosmic string with singularity.

It is also possible an another much radical interpretation. The smoothness of tensor fields is a natural requirement within the theory of differential spaces. In some sense the non smoothness is a symptom of the field theory consistency breaking. The theory of gravitational perturbations of the straight cosmic string's metrics with regularity conditions at the singularity [1,2,7] can be treated as a field theory on a d-space which is not a manifold. Therefore, it is natural to assume that the gravitational perturbations have to be smooth one on the whole cosmic string's space-time with singularity. Such assumption is equivalent to the Aliev and Gal'tsov condition of vanishing the conical bremsstrahlung effect and leads to the following discrete spectrum of the deficit of angle: $\Delta = 2\pi(1 - 1/n), n = 1, 2, 3, ...$

Acknowledgments

References

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